# ROTATIONALLY INVARIANT TEXTURE BASED FEATURES

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**ABSTRACT**

Content-based retrieval is ultimately dependent on the features used for the annotation of data and its ef£ciency is dependent on the invariance and robust properties of these features. For texture based features an important form of invariance is rotational invariance. In this paper novel rotationally invariant texture based features are introduced that are extracted from a Polar Fourier Transform (PFT). The PFT is similar to the Discrete Fourier Transform in two dimensions but uses transform parameters radius and angle rather than the Cartesian co-ordinates. The PFT is discretised appropriately across the angular and radial frequency space with the transform magnitudes forming the rotationally invariant features. These features although rotationally invariant, capture the angular distribution together with the radial distribution of frequency within texture. Preliminary results show the method to give better results than rotationally variant and invariant Gabor £lter schemes.

## 1. INTRODUCTION

The ability to effectively retrieve images or video according to their content is still an unful£lled goal for multimedia applications and therefore a currently active research area. Content-based retrieval is dependent on the features used for the annotation of data and its ef£ciency is dependent on the ability of extracted features to facilitate meaningful responses to a range of queries. For texture based features, this ability is to a large extent, dependent on the invariance and robust properties of the features. These properties include the invariance to scale, rotation, illumination transforms and robustness against noise. Inclusion of these properties should ensure that the features capture a more abstract representation of the texture separate from the circumstances in which it is found. This paper focuses on texture features that have been developed to be invariant to a transformation of a texture by rotation. Rotational invariance of texture features can be separated into two different classes: Isotropic and Anisotropic. Isotropic rotational invariant features are formed from averaged measures of some property (such as frequency content) in all directions. This is the most obvious way of obtaining rotationally invariant features. However in order to produce a richer representation the angular distribution of such a property can be characterised whilst remaining rotationally invariant. This type of rotational invariance is known as anisotropic.

## 2. REVIEW

Several methods have been proposed to extract rotationally invariant texture based features without using spatial-frequency analysis including circularly symmetric autoregressive (CSAR) models [1] and iterative morphological decompositions [2]. However, most attention has been focused on spatial-frequency analysis methods such as wavelet and pyramid decompositions as in general they have produced the best results. The steerable pyramid [3, 4, 5] is a spatial-frequency decomposition that has been utilised to produce isotropic and anisotropic rotationally invariant texture based features. It has the advantage over a normal wavelet type decomposition in that the output subbands can be adjusted to analyse many different orientations at each scale. Unfortunately this transform has the disadvantage of being considerably overcomplete with the amount of overcompleteness increasing with the number of analysed orientations. The more common dyadic wavelet decompositions used in many compression applications have also been used to extract texture features that are rotationally invariant. This has been done by simple combinations of subband energy measures [6] or by the modelling of the variation in the energy of each subband using Markov models [7]. These methods however lack the directional selectivity of the steerable pyramid. Wu and Wei [8] have used a classical dyadic wavelet decomposition on a spiral-resampling lattice. The phase (and therefore the rotation) of the spiral is removed in the decomposition thus enabling rotationally invariant measures to be produced from the resulting subbands. The £lter separability and iterative nature of the dyadic wavelet transform necessary for computationally ef£cient decomposition produces frequency analysis in octave bands. This may be considered to be too coarse for accurate texture analysis. Non-separable wavelets have been used [9] to produce more ¤exible isotropic and anisotropic rotationally invariant features over greater radial frequency ranges.

This paper is organised as follows. Initially the properties of the Polar Fourier Transform are investigated together

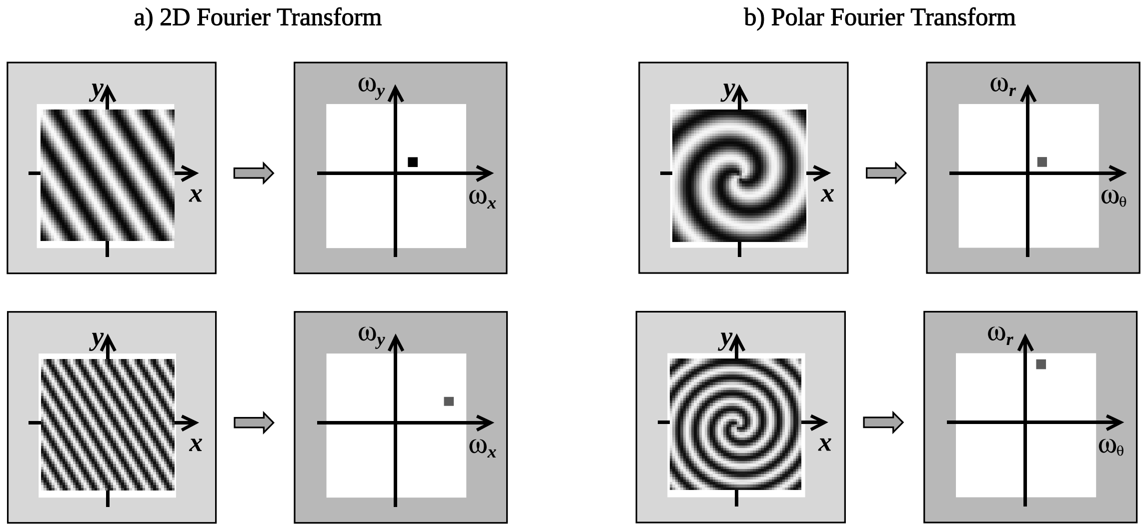


**Fig. 1**. Spatial visualisations of positions in FFT and PFT transform space

with visualisations of the analysis functions. The texture classi£cation experiments using the developed features are then described together with the experimental results.

## 3. THE POLAR FOURIER TRANSFORM (PFT)

The traditional discrete Fourier transform in two dimensions takes the form shown in equation 1. The parameters mapping the transform domain are the vertical and horizontal Cartesian frequency parameters and £gure 1a shows the real parts of several two dimensional spatial basis functions and their corresponding position in transform space. This transform domain can therefore be considered as analysing the frequency content in terms of such orientated complex sinusoidal gratings.

$$F(\omega_x, \omega_y) = \frac{1}{N^2} \sum_{x=0}^{N-1} f(x,y) exp\left[-j2\pi(\omega_x x + \omega_y y)/N\right] \quad (1)$$

The parameters that map the transform domain of the PFT are the frequency parameters relating to the angular and radial frequency i.e. $\omega_r$ and $\omega_\theta$ in equation 2. Points in the transform space therefore correspond to complex spiral-type basis functions as depicted in £gure 1b. The magnitude of the complex output of the PFT can be used to form rotationally invariant texture based features. However, when applied directly to the spatial domain, the PFT is not invariant to translations. The PFT transformation is only applied after an initial FFT is performed on the texture in order to remove translation dependence.

$$F(\omega_r, \omega_\theta) = \frac{1}{N^2} \sum_{x=0}^{N-1} f(x,y) exp\left[-j(\pi \times \omega_r r/N + \omega_\theta \theta)\right] \quad (2)$$

The features are therefore extracted as represented by £gure 2. i.e. the features are formed from the magnitudes of the PFT output after an FFT transformation. When discretising the parameters, $\omega_\theta$ only needs to take even values as the frequency plane is symmetric. The discretisation of the $\omega_r$ parameter is discretionary but would be expected to produce the best results when considering an even or logarithmic spread. The best results were in fact produced from an empirically derived and small set (see section 4). These features are not only rotationally invariant but also characterise the angular frequency distribution of the texture i.e. they have anisotropic rotational invariance. A similar method is used in pattern recognition called the Fourier-Mellin transform [10] where the rotational invariance is achieved by resampling on a log-polar lattice in between two FFT transforms. However, developing texture features obtained directly from the PFT obviates the need for the interpolation necessary for the translation to log-polar (or just polar) coordinates. This interpolation may be considered to introduce unnecessary errors due to the necessary averaging operations and also the variations in coverage for different radii.

## 4. EXPERIMENTAL RESULTS

Sixteen textures were taken from the Brodatz texture album [11] to test the classi£cation performance of the developed features. These textures (shown in £gure 3) were

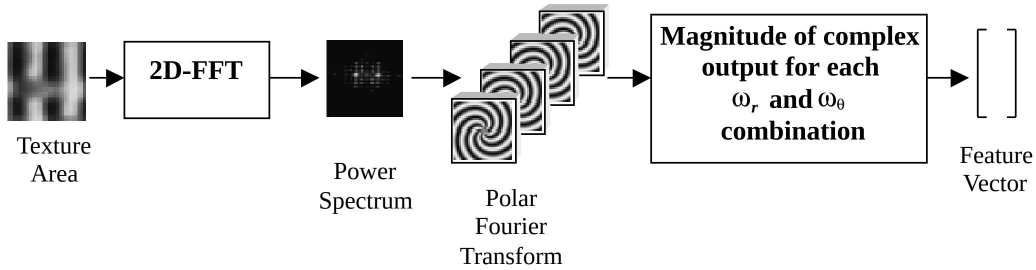


**Fig. 2**. Extraction of texture features from the Polar Fourier Transform

chosen to represent textures that contained a range of periodic, stochastic and directional elements. The textures were scanned as eight-bit raw grey level images of size $256 \times 256$ pixels.

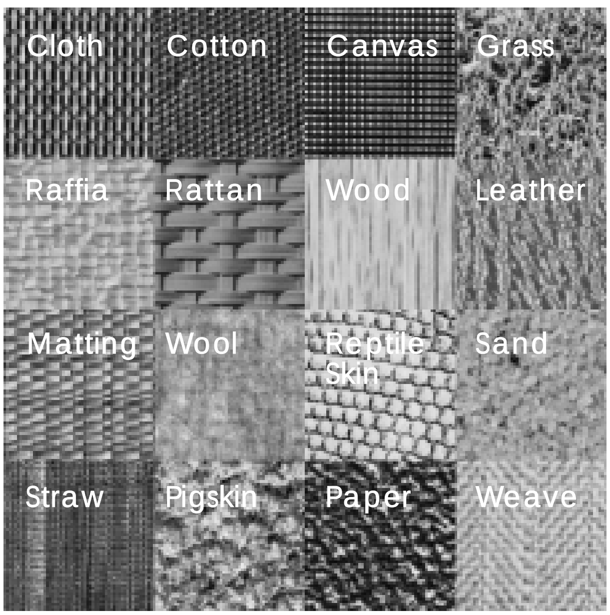


**Fig. 3**. 16 Brodatz textures used in texture classi£cation experiments

One version of each texture class was used for training at angles of $0°, 30°, 45°$ and $60°$. Seven different versions of each texture were used for classi£cation and presented at angles $20°, 70°, 90°, 120°, 135°$ and $150°$. This gave 42 classi£cations per texture and 672 in all. Each training texture was tiled into $16 \times 16$ squares with feature values being extracted from the PFT decomposition of each tile leading to a mean vector and covariance matrix for each texture class. The four angles of training were used to enable the mean feature vector and the covariance matrix to be properly estimated under texture rotation. Similarly, the mean feature vectors were extracted from the test textures from the PFT transform of tiled $16 \times 16$ squares. Textures were classi£ed using a minimum Mahalonobis distance classi£er. Table 1 shows the results for the most ef£cient parameter set for the PFT ($\omega_r = \{0.0, 0.5, 1.7, 3.0\}$, $\omega_\theta = \{0.0, 2.0, 4.0, 6.0\}$, i.e. combined to give 16 features in all). An exhaustive number of alternative sets were also considered. However this empirically determined set was found to produce the best results. It was assumed that less parameters would fail to properly map the polar Fourier transform domain and that more would suffer from aliasing. Table 1 also compares these results with other comparable methods. The rotationally variant Gabor £lters are based on those developed by Porter [6] and show the reduced ef£ciency of these traditional texture based features when applied to the classi£cation of rotated textures. The (isotropic) rotationally invariant Gabor features (also developed by Porter) show an improvement but are still inferior to the PFT features. This is assumed to re¤ect that invariant Gabor features do not characterise the angular distribution of the textures within their de£ned annular frequency bands.

| **Feature Extraction Technique** | **Correct Classi£cation Rate (%)** |
|---|---|
| **PFT derived features at each scale** | 89.58 |
| **Rotationally variant Gabor £lters** | 70.53 |
| **Rotationally invariant Gabor £lters** | 82.89 |

**Table 1**. Classi£cation performance of wavelet features on rotated images: decomposition on $16 \times 16$ areas, best performing feature sets for each method.

## 5. CONCLUSION

Traditional texture based features for retrieval often do not exhibit rotational invariance as can be seen from the results for the rotationally variant Gabor £lters in table 1. Conversely, many rotationally invariant texture based features do not characterise the angular distribution of frequency content within the texture i.e. they have isotropic rotational invariance. The features here developed from the PFT are not only rotationally invariant but also characterise the angular frequency content of the texture i.e. they have anisotropic rotational invariance. However, one disadvantage of using the magnitude of the PFT output (as described above) is the removal of radial phase information that would provide extra useful information for classi£cation. Indeed although the PFT magnitude output does produce rotationally invariant features in general it does not capture an intuitively understandable representation of frequency content. This may not prove to be a disadvantage but may have implications for content based retrieval of natural images where there will be a signi£cantly higher number of textures to distinguish. It does however effectively characterise the frequency content of the image / texture in a rotationally invariant fashion as is demonstrated in the above results. The classi£cation performance in the conducted tests of a feature vector formed from rotational harmonics extracted from a PFT decomposition was over 5% better than an alternative rotationally invariant method based on circularly symmetric Gabor £lters. Using the PFT within this type of method may be extended into an analogous transform of the Fourier-Mellin transform where by the axes of the PFT are not just polar but log polar. Theoretically this should produce scale and rotationally invariant texture based features. This should be a valuable extension to this work.

Experience has shown that classi£cation results can be extremely dependant on the choice of test conditions and image set. It is therefore dif£cult to claim a de£nitive improvement on previous rotational invariant characterisation methods. However the PFT method provides a ¤exible approach for generating aniostropic rotationally invariant features that could be potentially integrated with scale invariant features.

## 6. REFERENCES


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